ARMY RESEARCH LABORATORY

Analytical Blast Model Formulation With Computer Code

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Abstract

Overpressure time history data from warhead blast experiments yield peak overpressure P as a function of spatial position.

Dr. Owen Litt has proposed a model for P based on the peak-overpressure characteristics of a bare spherical charge. The direction-independent peak-overpressure function for a bare spherical charge is modified to have nonspherical level-surface structure by specifying surfaces of constant peak overpressure. This introduces a directional component into the model.

In this report, the original formulation is refined and generalized and a mathematical model and computer code are presented to evaluate the function. Such a computational device is required for model parameter estimation and experiment design.

Acknowledgments

Thanks are extended to Ed Davisson for numerous enlightening discussions regarding mathematical modeling and the philosophy of mathematics in general. Andrew Thompson also reviewed the paper and provided guidance for improving the overall presentation of the topic. Without his input, this paper would certainly be less readable. Finally, thanks go to thank Dr. Owen Litt for sharing his concept of the peak-overpressure model.

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1. Background

Overpressure time history data from warhead blast experiments yield peak overpressure P as a function of spatial position. Dr. Owen Litt [1] has proposed a model for P based on the peak-overpressure characteristics of a bare spherical charge. In that model, the direction-independent peak-overpressure function for a bare spherical charge is modified to have a level curve structure with a specific nonspherical functional form. This induces a directional component into the model.

It is necessary to combine the peak-overpressure function representations of the bare spherical charge and an arbitrary level-curve structure to produce the required mathematical model. This report details the solution of that analytical problem and also an explicit solution to the problem of level-curve model specification in general, and so serves a twofold purpose. The development of a general theoretical framework for solving such model-specification problems appears in section 2. The rest of this report describes the application of the general principle to the specific problem of Dr. Litt's blast model. The original formulation is refined and generalized and a mathematical model and computer code are presented to evaluate the function.

2. Level-Curve Model Specification

This section contains a discussion of model formulation based on specifying a geometric level-curve structure. An example precedes the development of a general method for the formulation of such models.

2.1 An Example. Suppose F is a decreasing function on $[0, \infty)$. For example, take

$$F(r) = \frac{1}{1+r^2}. \tag{1}$$

The function F can be used to create function $F_1 : \mathbb{R}^2 \to \mathbb{R}^+$ by defining

$$F_1(r, \phi) = F(r) = \frac{1}{1+r^2}, \tag{2}$$

where the usual Cartesian coordinates on $\mathbb{R}^2$ are (x, y) and polar coordinates (r, ϕ) on $\mathbb{R}^2$ are given by $x = r\cos\phi$ and $y = r\sin\phi$. In the x-y plane, level curves of F_1 are concentric circles centered at the origin, since F_1 is independent of ϕ. For any $L \in (0, 1]$, the value of r that makes $F_1(r, \phi) = L$ is given by $F_1(r, \phi) = F(r) = 1/(1+r^2) = L$, so that $r = F^{-1}(L) = \sqrt{1/L - 1}$. In other words, $F(\sqrt{1/L-1}) = L$, so $F_1 = L$ on the circumference of a circle with radius $\sqrt{1/L-1}$ and area $\pi(1/L - 1)$. It is possible to modify the definition of F_1 and produce a function F_2 that has elliptical level curves with a given eccentricity and

orientation. Furthermore, the value of F_1 on a given circle should be equal to the value of F_2 on an ellipse with the same area. The polar equation for an ellipse is

$$r^2 = ab \cdot \sqrt{1-\varepsilon^2} \cdot \frac{1+\tan^2(\phi-\phi_o)}{1-\varepsilon^2+\tan^2(\phi-\phi_o)}, \tag{3}$$

where the ellipse has eccentricity $\varepsilon = \sqrt{1-b^2/a^2}$, major axis length $2a$ in the direction $\phi = \phi_o$, minor axis length $2b$ in the direction $\phi = \phi_o + \pi/2$, and area πab. The function F_2 is now defined by

$$F_2(r,\phi) = F\left(r \cdot \left[\sqrt{1-\varepsilon^2} \cdot \frac{1+\tan^2(\phi-\phi_o)}{1-\varepsilon^2+\tan^2(\phi-\phi_o)}\right]^{-1/2}\right). \tag{4}$$

Then, it can be seen that $F_2(r,\phi) = L$ when

$$r^2 = \left(\frac{1}{L}-1\right) \cdot \sqrt{1-\varepsilon^2} \cdot \frac{1+\tan^2(\phi-\phi_o)}{1-\varepsilon^2+\tan^2(\phi-\phi_o)}, \tag{5}$$

so that $F_2(r,\phi) = L$ on the perimeter of an ellipse of area $\pi(1/L-1)$. (See Figures 1 and 2 for a depiction of the level curves of these example functions.)

2.2 The General Construction. Consider a decreasing function $F\colon [0,\infty) \to \mathbb{R}^+$. This function F can be used to create a function $F_1\colon \mathbb{R}^2 \to \mathbb{R}^+$ by defining

$$F_1(r,\phi) = F(r), \tag{6}$$

where r and ϕ are polar coordinates. Since F_1 is independent of ϕ, the level curves of F_1 are concentric circles centered at the origin of $\mathbb{R}^2$. And because F is decreasing, the value of F_1 is smaller on a larger such circle.

It is possible to construct a version of F that has noncircular level curves with any specific functional form. In particular, say the level curves are to be given by

$$r(\phi) = g_\phi(u) \tag{7}$$

for various values of u. Suppose for all ϕ that $g_\phi(u)$ is a continuous function of u, that $g_\phi(0) = 0$, that $g_\phi(u)$ is an increasing function of u, that $g_\phi(u)$ is defined for all $u \geqslant 0$, and that $\lim_{u\to\infty} g_\phi(u) = \infty$. So, g_ϕ is a bijective function on $[0,\infty)$, and g_ϕ has an inverse in the following sense: for each fixed value of ϕ, the inverse function g_ϕ^{-1}, characterized by $g_\phi\left(g_\phi^{-1}(u)\right) = g_\phi^{-1}\left(g_\phi(u)\right) = u$, is well-defined for $u \geqslant 0$. In fact, $g_\phi^{-1}(u)$ is also an increasing function of u on $[0,\infty)$.

A function $F_2\colon \mathbb{R}^2 \to \mathbb{R}^+$ satisfying equation (7) can be defined in terms of polar coordinates by

$$F_2(r,\phi) = F\left(g_\phi^{-1}(r)\right). \tag{8}$$

To show this, let $u > 0$ be constant. Then $F(u)$ is also constant, and the locus of (r,ϕ) which has $F_2(r,\phi) = F(u)$ is given by $F(u) = F_2(r,\phi) = F\left(g_\phi^{-1}(r)\right)$. This means that $u = g_\phi^{-1}(r)$, from which equation (7) follows.

3. Application to Blast Model Formulation

Here, the results of the previous section are applied to the formulation of models for the maximum peak overpressure of a detonating charge blast field. First, the basic formulation is discussed and then an enhanced model is presented.

3.1 Basic Model. This model works in two-dimensional polar coordinates (r, ϕ) with the origin centered on the detonating charge. A three-dimensional spatial model for peak overpressure P can be obtained by rotating a two-dimensional model, defined in the half-plane $0 \leqslant \phi \leqslant \pi$, about the x-axis.

The development of a two-dimensional model for peak overpressure as a function of the polar coordinates (r, ϕ) follows. In this report, models for peak overpressure are based on the function $P_s(z)$, which gives the maximum peak overpressure for detonation of a spherical TNT charge. In the definition of P_s and throughout this report, the normalized distance coordinate

$$z = \frac{r}{W^{\alpha}} \tag{9}$$

is used, where r is measured in feet, W is charge weight in pounds, and α is a constant with nominal value $\alpha = 1/3$. The spherical charge pressure function $P_s(z)$ itself is defined by

$$P_s(z) = \exp\left[\frac{A}{z+B} - C\right], \tag{10}$$

where the constants have the empirical values $A = 31.97$, $B = 3.555$, and $C = 0.5$. This function was derived from a fit to empirical data [1].

The modeling concept under consideration requires a pressure function component P_n with a nontrivial dependence on ϕ, specified by a certain family of noncircular level curves. The function P_n is derived from P_s in the same way that F_2 is derived from F in section 2, by the application of equation (8) to a specific level curve function. The level curve function for P_n, which gives an appropriate shape based on engineering considerations, is given by

$$g_{\phi}(u) = u\left[\sin\frac{m\phi}{2}\right]^{4n(u)/m}, \tag{11}$$

where $n(u)$ is defined shortly, and P_n is defined by

$$P_n(z, \phi) = P_s\left(g_{\phi}^{-1}(z)\right), \tag{12}$$

as in section 2.

Two more definitions complete the specification of P_n. The function f is defined to be a "smooth step function" with $f(0) \simeq 0$, f increasing, and $f(z) \rightarrow 1$ as $z \rightarrow \infty$. To be specific, set $z_o = 10$ and take

$$f(z) = \frac{1}{2} + \frac{1}{\pi} \arctan(z - z_o). \tag{13}$$

The function n is defined by

$$n(u) = n_o\big(1 - f(u)\big), \tag{14}$$

where n_o is a positive constant. So n is a decreasing function with $n(0) = n_o$ and $n(u) \rightarrow 0$ as $u \rightarrow \infty$. The behavior of n along with the form of g_ϕ implement the design objective that P_n looks like P_s at large distances; i.e., the level curves of P_n become circular for large z.

Now with the function P_n completely specified, the conditions on $g_\phi(u)$ of section 2 are indeed satisfied. The exponent $4n(u)/m$ is positive and decreasing with u, so $\sin(m\phi/2)^{4n(u)/m}$ is a nondecreasing function of u for fixed ϕ. Therefore, $g_\phi(u)$ is increasing in u for any ϕ. The conditions of section 2 are satisfied, so a level curve of $P_n(z,\phi)$ is given by $z = g_\phi(u)$ for u fixed, as required.

To specify the peak overpressure model, it remains only to combine the function components P_s and P_n in a certain way. The definition of the peak overpressure model function P is

$$P(z,\phi) = f(z)P_s(z) + \big(1 - f(z)\big)P_n(z,\phi), \tag{15}$$

where the functions P_s, P_n, and f are as previously discussed. Because of the nature of f, the pressure function looks like P_n for small z and like P_s for large z. Definition of the model is now complete. The quantities P, z, f, P_s, g_ϕ, and n were specified by Dr. Owen Litt [1, 2, 3, 4, 5], as was an implicit characterization of P_n. The explicit representation of equation (12) for P_n is a product of this report.

In summary, the complete model is given by

$$\begin{aligned} P(z,\phi) &= f(z)P_s(z) + \big(1 - f(z)\big)P_n(z,\phi), \quad \text{where} \\ z &= r/W^\alpha, \\ f(z) &= 1/2 + 1/\pi \cdot \arctan(z - z_o), \\ P_s(z) &= \exp\big(A/(z+B) - C\big), \\ n(u) &= n_o\big(1 - f(u)\big), \\ g_\phi(u) &= u\,(\sin m\phi/2)^{4n(u)/m}, \quad \text{and} \\ P_n(z,\phi) &= P_s\big(g_\phi{}^{-1}(z)\big). \end{aligned} \tag{16}$$

The quantities A, B, C, and W are constant; $z_o = 10$, $z_1 = 5$, and $z_2 = 15$ are fixed model parameters; and m, n_o, and α are model parameters to be estimated. Interpretations of

the parameters are as follows: m determines the direction of the P_n component, the value of n_o makes the P_n component more or less concentrated in the direction determined by m, and α determines the dependence of normalized distance on charge weight.

Now some characteristics of the model can be examined in more detail. The extreme point on a level curve occurs when $\phi = \pi/m$ and $\sin(m\phi/2) = 1$, in which case $g_{\pi/m}(u) = u$ and also $g_{\pi/m}^{-1}(u) = u$. Then, $P_n(z, \pi/m) = P_s(z)$. So, in the direction of maximum peak overpressure, $\phi = \pi/m$, the P_n component has the same pressure value as spherical bare charge, P_s. In other directions, for fixed z, the value of P_n is lower than P_s.

For an example, set the charge weight to $W = 1$ and set the function parameters to $m = 1.75$, $n_o = 2.0$, and $\alpha = 1/3$. Figure 3 demonstrates the level curve characterizations of P_n and P_s at the same pressure value. P_n is evaluated at the point $(z_o, \phi_o) = (4, \pi/3)$. This point lies on the level curve $z = g_\phi(u)$, where $g_{\phi_o}(u) = z_o$, or $u = g_{\phi_o}{}^{-1}(z_o)$, so a general point on this level curve has coordinates $(g_\phi(u), \phi)$. The extreme point on this level curve, where $\phi = \pi/m$, has coordinates $(g_{\pi/m}(u), \pi/m) = (u, \pi/m)$. The value of P_n anywhere on this level curve is $P_n((g_\phi(u), \phi)) = P_s(g_\phi{}^{-1}(g_\phi(u))) = P_s(u)$. Particular values for this example are $u \simeq 8.94$ and $P_s(u) \simeq 8.49$.

Once again, in the direction of maximum peak overpressure, $\phi = \pi/m$, the P_n component has the same pressure value as spherical bare charge, P_s. In other directions, for fixed z, the value of P_n is lower than P_s. This may not be realistic. In the maximum direction, P_n should have a higher value than P_s, since the blast modeled by P_n is focused in that direction. This additional feature is implemented by incorporating into the definition of P_n an equivalent spherical charge weight W_s that is greater than the actual charge weight W, effectively renormalizing the distance coordinate in equation (9), which is then used in equations (10) and (12). A conceptually equivalent approach is to directly reduce the distance argument z of P_s in equation (10), as it is used in the definition of P_n. Alternatively, the level curve function can be changed in the definition of P_n from g_ϕ, equation (11), to a new function produces a higher pressure value on the level curve.

As shown in section 3.2, these three schemes are equivalent. The net effect of any of them is to force a P_n level curve to correspond to a smaller P_s level curve, on which the pressure is higher. The basic model can be modified to have this property.

3.2 Enhanced Model. The model of the section 3.1 is generalized by the introduction of a new pressure function P_n^*, which replaces P_n. The P_n^* component in the maximum direction ($\phi = \pi/m$) has the pressure value of a spherical charge of arbitrary weight $W_s(z)$. To increase the generality and flexibility of the model, W_s is allowed to be a function of z rather than a constant. It is convenient to define the function M by

$$M(z) = \frac{W_s(z)}{W}, \tag{17}$$

so that M represents a dimensionless mass scaling ratio or magnification factor in the maximum direction, since $W_s(z)$ is the equivalent sphere charge weight in that direction

(π/m) at the distance $r = zW^{\alpha}$. Now, define P_n^* by

$$P_n^*(z,\phi) = P_s\left(s\left(g_{\phi}^{-1}(z)\right)\right) \tag{18}$$

and proceed to solve for s. When $\phi = \pi/m$, the result is

$$P_n^*(z,\pi/m) = P_s\left(s\left(g_{\pi/m}^{-1}(z)\right)\right) = P_s\left(s(z)\right). \tag{19}$$

Since $z = rW^{-\alpha}$ and $s(z) = rW_s(z)^{-\alpha} = rW^{-\alpha}M(z)^{-\alpha}$, it follows that $r = zW^{\alpha} = s(z)W^{\alpha}M(z)^{\alpha}$, and then

$$M(z) = \left[\frac{z}{s(z)}\right]^{1/\alpha} \quad \text{or} \quad s(z) = zM(z)^{-\alpha}. \tag{20}$$

This expresses the required function s in terms of the magnification factor M and, therefore, also in terms of the equivalent sphere weight W_s. Note, referring to equation (10), that the function s as it appears in $P_s(s(z))$ amounts to a rescaling of distance in the function P_s. Also, the definition of P_n^* can be written as $P_n^*(z,\phi) = P_s((g_{\phi}\circ h)^{-1}(z))$, where $h^{-1} = s$, and thus making explicit the modification of the level curve function in the definition of P_n^*. So the three conceptual approaches (weight scaling, distance scaling, and level curve modification) to the derivation of P_n^* from P_n are equivalent.

Now, with $W_s(z) \equiv W$, then $M(z) \equiv 1$ and $s(u) = u$, so that $P_n^* = P_n$. This reduces to the basic model of the section 3.1, where the P_n has the property that, in the maximum pressure direction $\phi = \pi/m$, the peak overpressure is equal to that of a spherical charge of the same weight.

A more realistic general formulation requires that $M(0) > 1$, that $M(z)$ is a nonincreasing function of z, and that $M(z) \to 1$ as $z \to \infty$. This behavior embodies the design criteria that P_n^* itself looks like P_s at large distances and that the pressure P_n^* is greater than P_s in the maximum direction at small distances.

It may be possible to completely specify M through energy conservation considerations, but, for illustrative purposes, a piecewise continuous version of M is used. This has corresponding s that is easy to calculate. Let $M(z) = M_o > 1$ for $z < z_1$, let $M(z) = 1$ for $z > z_2$, and let $M(z)^{\alpha}$ be a linear function of z for $z_1 \leqslant z \leqslant z_2$. It is convenient to express piecewise function definitions in terms of the indicator function

$$I_T(t) = \begin{cases} 1, & t \in T \\ 0, & t \notin T. \end{cases} \tag{21}$$

First define the "linear step function" L with the characteristics that $L(z) = b$ for $z \leqslant z_1$, $L(z) = 1$ for $z \geqslant z_2$, $L(z)$ is linear for $z_1 \leqslant z \leqslant z_2$, and L is continuous. The appropriate definition is

$$L(z;\, b, z_1, z_2) = b \cdot I_{[0,z_1)}(z) + (a_1 + a_2 z)\cdot I_{[z_1,z_2]}(z) + 1 \cdot I_{(z_2,\infty)}(z), \tag{22}$$

$$\text{where} \quad a_1 = \frac{bz_2 - z_1}{z_2 - z_1} \quad \text{and} \quad a_2 = \frac{1-b}{z_2 - z_1}. \tag{23}$$

The corresponding definition for M is then

$$M(z) = L(z;\, M_o^\alpha, z_1, z_2)^{1/\alpha}. \tag{24}$$

Solve for s in closed form to get

$$s(z) = z/b \cdot I_{[0,z_1)}(z) + z/(a_1 + a_2 z) \cdot I_{[z_1,z_2]}(z) + z \cdot I_{(z_2,\infty)}(z), \tag{25}$$

where $b = M_o^\alpha$, and a_1 and a_2 are given by equation (23).

A complete working model is then

$$\begin{aligned} P(z,\phi) &= f(z)P_s(z) + \big(1 - f(z)\big)P_n^*(z,\phi) \quad \text{where} \\ z &= r/W^\alpha, \\ f(z) &= 1/2 + 1/\pi \cdot \arctan(z - z_o), \\ P_s(z) &= \exp\big(A/(z+B) - C\big), \\ n(u) &= n_o\big(1 - f(u)\big), \\ g_\phi(u) &= u\,(\sin m\phi/2)^{4n(u)/m}, \\ M(z) &= L(z;\, M_o^\alpha, z_1, z_2)^{1/\alpha}, \\ s(z) &= zM(z)^{-\alpha}, \quad \text{and} \\ P_n^*(z,\phi) &= P_s\Big(s\big(g_\phi^{-1}(z)\big)\Big). \end{aligned} \tag{26}$$

The quantities A, B, C, and W are constant; $z_o = 10$, $z_1 = 5$, and $z_2 = 15$ are fixed model parameters; and m, n_o, M_o, and α are model parameters to be estimated.

The example of section 3.1 illustrates the enhanced model. Again, the charge weight is $W = 1$ and set the function parameters are $m = 1.75$, $n_o = 2.0$, and $\alpha = 1/3$. The new function parameter for mass scaling is $M_o = 4$. Figure 4 depicts the functions M and s. Figure 5 demonstrates the level curve characterization of P_n^* in relation to that of P_s at the same pressure. As before, P_n^* is evaluated at the point $(z_o, \phi_o) = (4, \pi/3)$. This point lies on the level curve $z = g_\phi(u)$ where $g_{\phi_o}(u) = z_o$, or $u = g_{\phi_o}{}^{-1}(z_o)$, so a general point on this level curve has locus $(g_\phi(u), \phi)$. The extreme point on this level curve, where $\phi = \pi/m$, has coordinates $(g_{\pi/m}(u), \pi/m) = (u, \pi/m)$.

Note that the P_n^* level curve is identical to the P_n level curve in Figure 3, which illustrates the example of section 3.1. The function value is different, however, to reflect the increased equivalent sphere charge weight or reduced distance in P_s. The corresponding P_s level curve in Figure 5 has a radius smaller than the extreme distance on the P_n^* level curve. The value of P_n^* anywhere on its level curve is $P_n^*((g_\phi(u), \phi)) = P_s(s(g_\phi^{-1}(g_\phi(u)))) = P_s(s(u))$, which is also the value of P_s on its level

curve in Figure 5. Particular values for this example are $u \simeq 8.94$, $s(u) \simeq 6.59$, and $P_s(s(u)) \simeq 15.6$. The weight scaling factor is

$$M(u) = \left[\frac{u}{s(u)}\right]^{1/\alpha} \simeq 2.49, \tag{27}$$

and, since $W = 1$, this is also the equivalent sphere charge weight in the direction $\phi = \pi/m$ at the distance $r = u$. These values of u, $s(u)$, and $M(u)$ are distinguished in Figure 4.

4. Model Evaluation for Experiment Design

Conducting an experiment to calibrate the model (estimate the parameters) involves placing pressure sensors in the detonation field of an explosive charge. The sensors must be placed so that optimal useful information is obtained from the experiment. Sensors cannot be overdriven. On the other hand, each sensor has a lower limit of resolution, beyond which the noise in the measurement system overrides any signal. Sensors must also be placed so that they register the nonspherical P_n component of the pressure field. It is therefore reasonable to "guess" what the model parameters are, evaluate P, and place the sensors accordingly.

A graphical display of the model response is useful in the design of an experiment for blast model parameter estimation. Since the model is a well-defined function, evaluation is conceptually simple: replace constants with numbers and evaluate the functions.

The form chosen for $M(z)$ yields a closed-form representation for s. Generally, g_ϕ^{-1} must be evaluated numerically, even if s has a closed-form representation. Choosing another form for $M(z)$ may result in s having no closed-form representation, which will increase computational complexity.

Figures 6–10 are contour representations of P computed with parameter values $W = 1$, $m = 1.75$, $n_o = 2.0$, $M_o = 4$, and $\alpha = 1/3$ on an various $x - y$ grids. Spatial coordinates are equivalent to z units, since $W = 1$. Contour levels are indicated in the figure captions. Note that the logarithmic spacing of the level curves gives a better visual display than linear spacing would. Figure 6 has $-20 \leqslant x \leqslant 20$ and $0 \leqslant y \leqslant 20$ to show the far field, Figure 10 has $-1 \leqslant x \leqslant 1$ and $0 \leqslant y \leqslant 1$ to show the near field, and the intervening figures depict intermediate ranges. Computations and graphics were done with *Mathematica* [6]; the code necessary to reproduce these calculations are presented in the Appendix.

5. Model Parameter Estimation

Data consist of empirical measurements of peak overpressure p at spatial location (r, ϕ), denoted as p_i, r_i, and ϕ_i for $1 \leqslant i \leqslant N$. Parameter estimates can be obtained, for example,

by least squares in the response or log response; i.e.,

$$\text{minimize} \sum_{i=1}^{N} [p_i - P(W^{-\alpha} r_i, \phi_i)]^2, \tag{28}$$

or

$$\text{minimize} \sum_{i=1}^{N} [\log p_i - \log P(W^{-\alpha} r_i, \phi_i)]^2, \tag{29}$$

where the minimizations are conducted over the parameter vector (m, n_o, M_o, α). Due to the exponential nature of P and the error characteristics of pressure sensors, estimation based on $\log P$ will most likely yield more accurate results than estimation based on P.

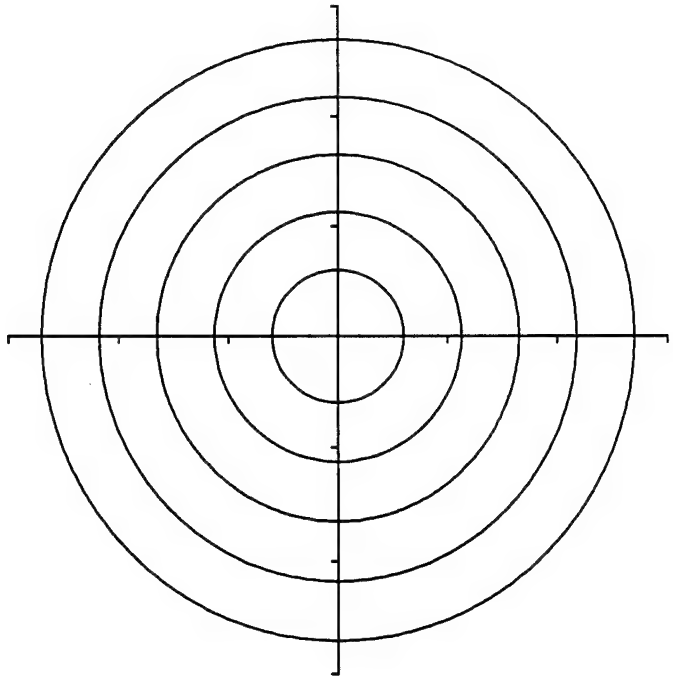

Figure 1. Level Curves of F_1 From Section 1.

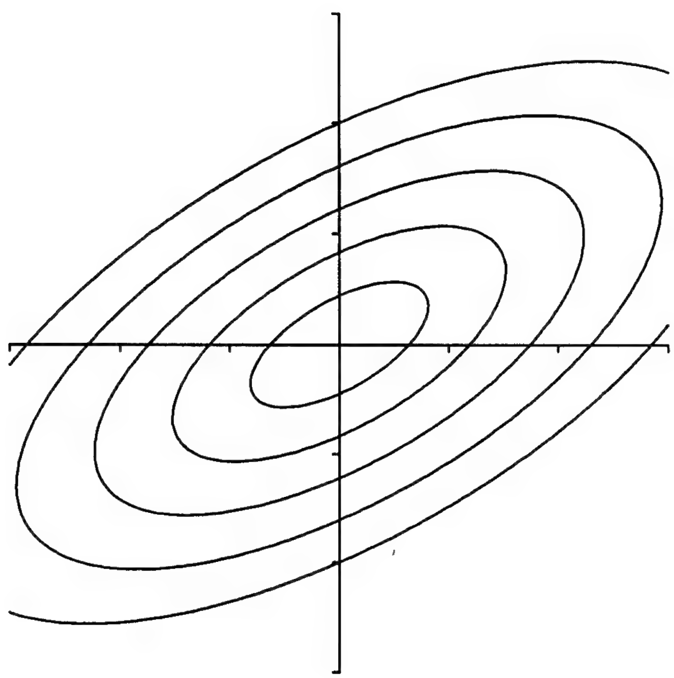

Figure 2. Level Curves of F_2 From Section 1.

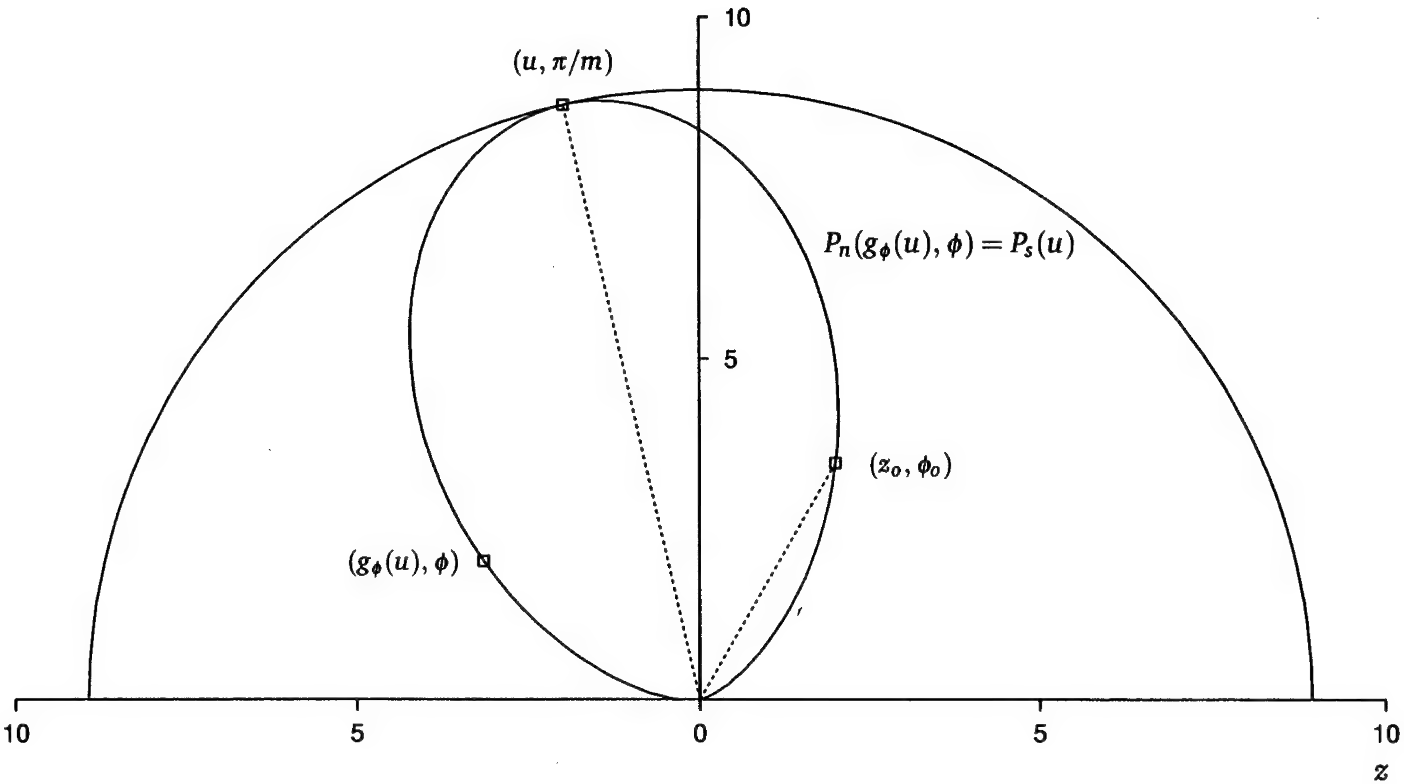

Figure 3. Level Curves for Pressure Components P_s and P_n.

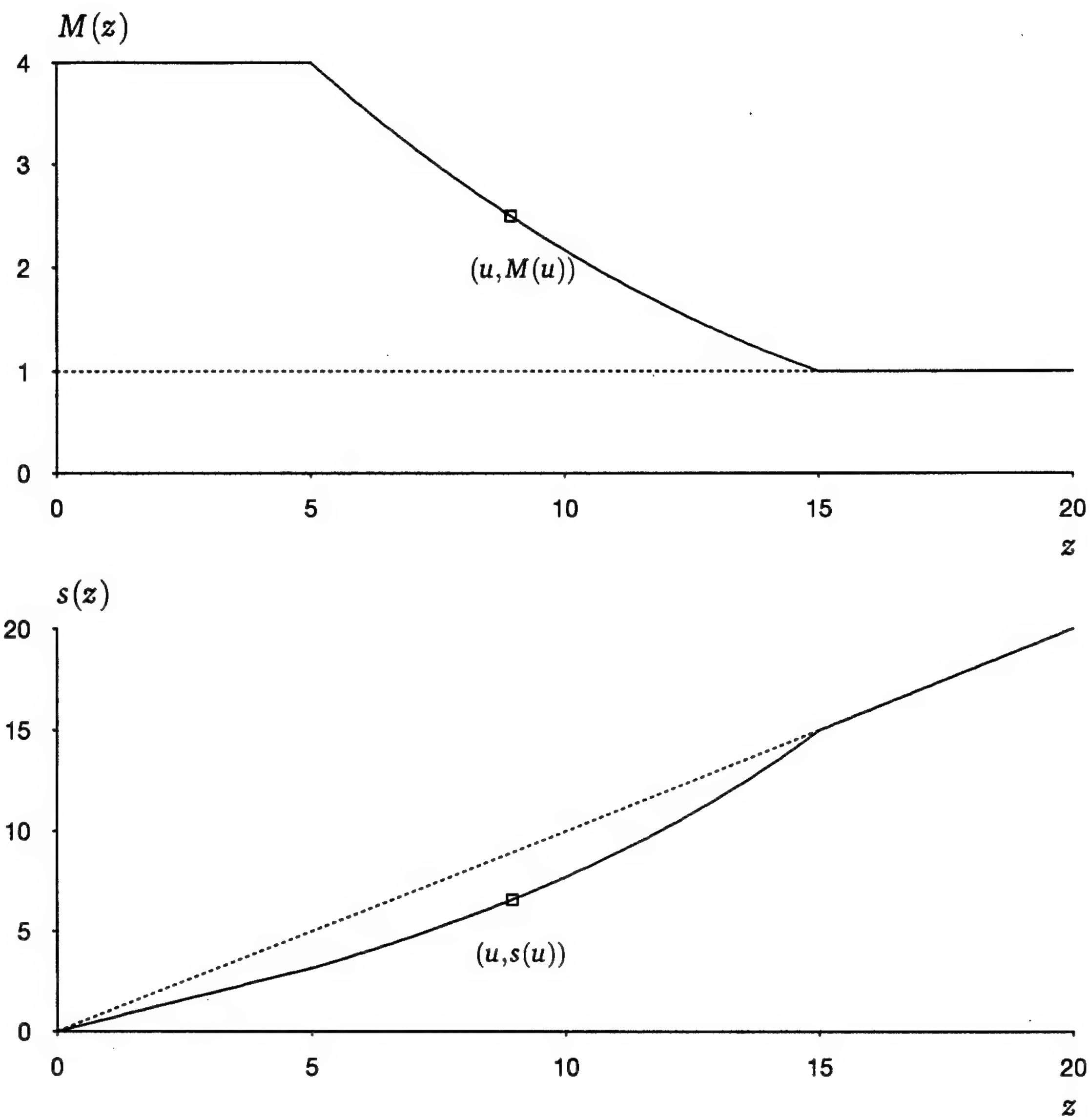

Figure 4. Mass Scaling Functions M and s.

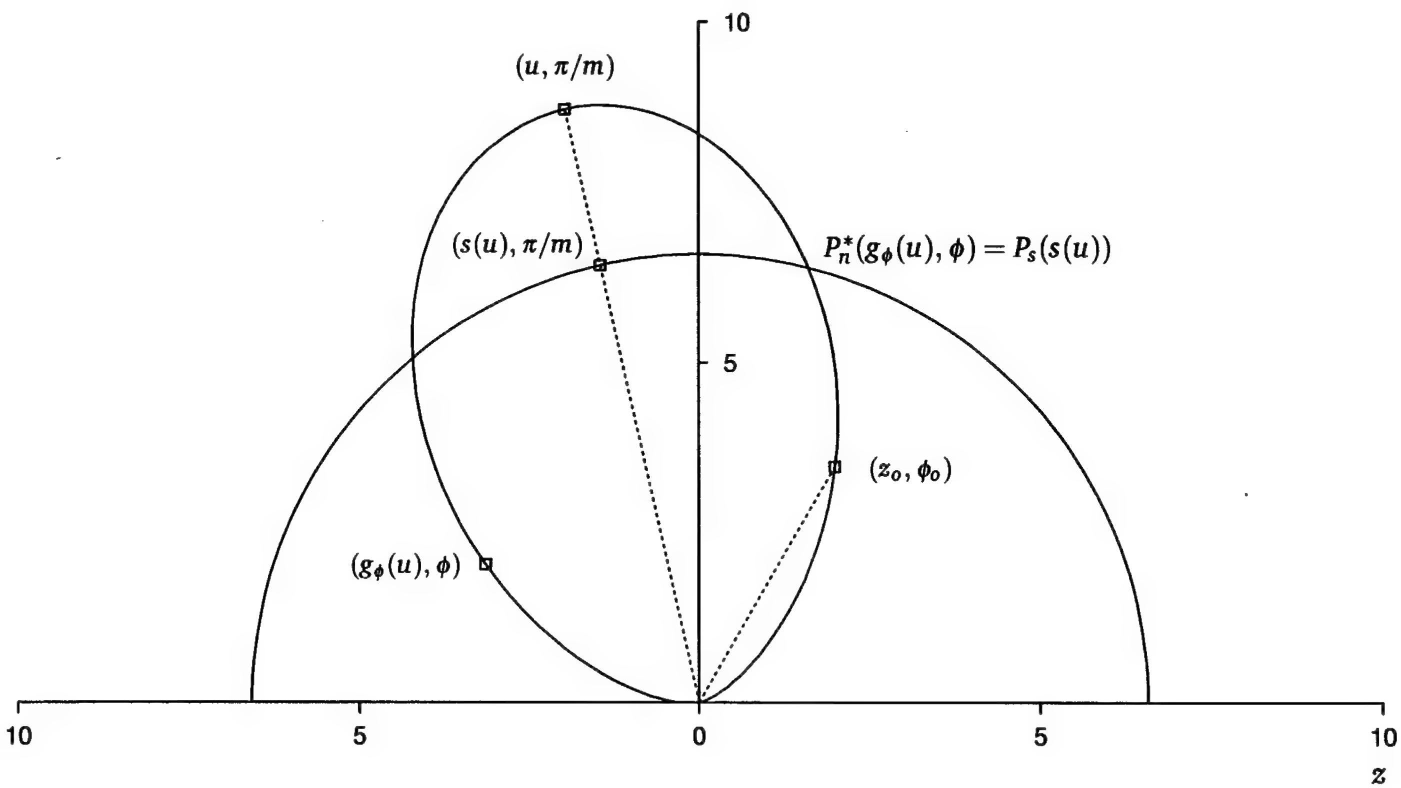

Figure 5. Level Curves for Pressure Components P_s and P_n^*.

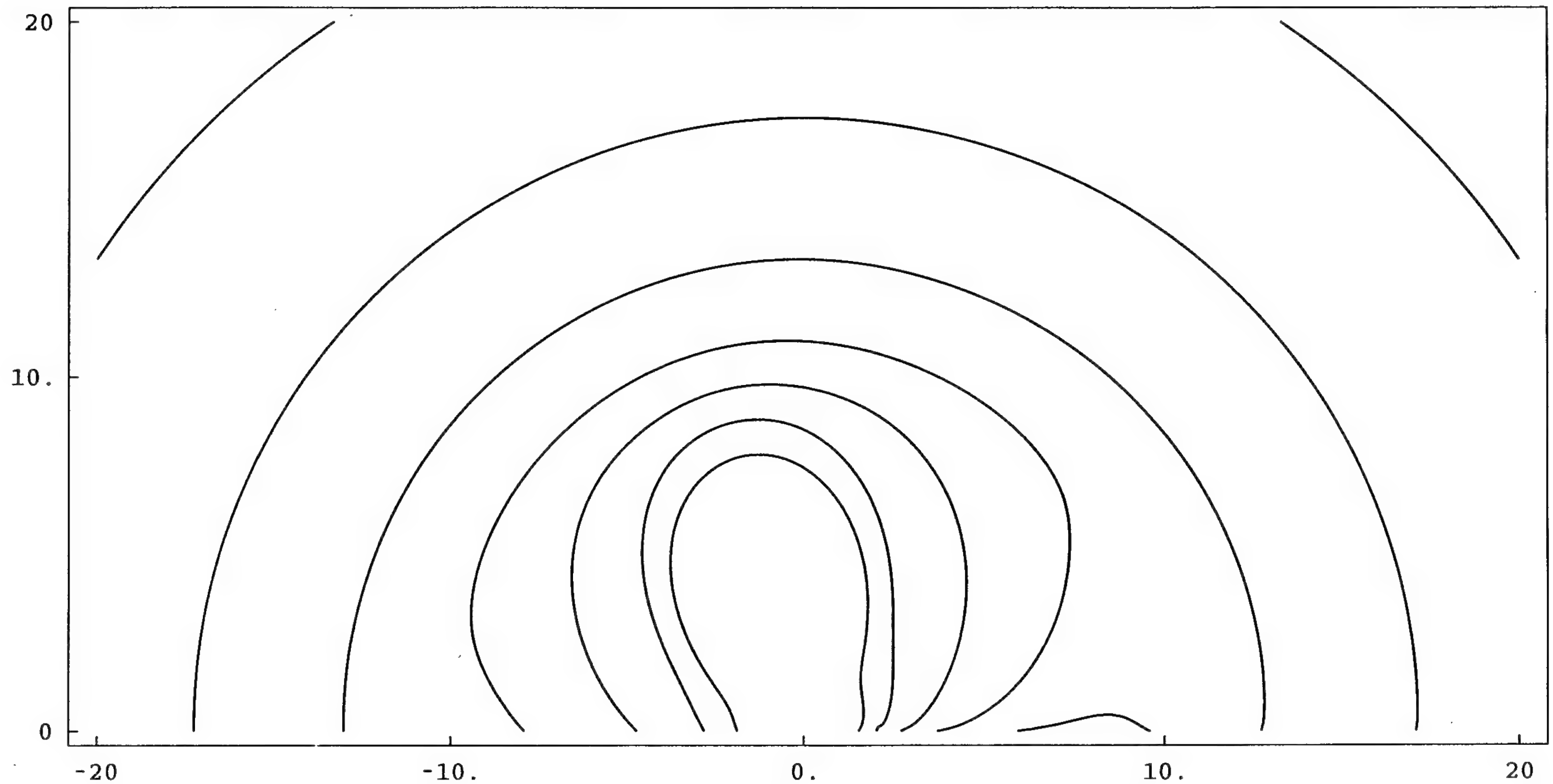

Figure 6. ***P*** **at (2, 3, 4, 6, 9, 14, 20) psi.**

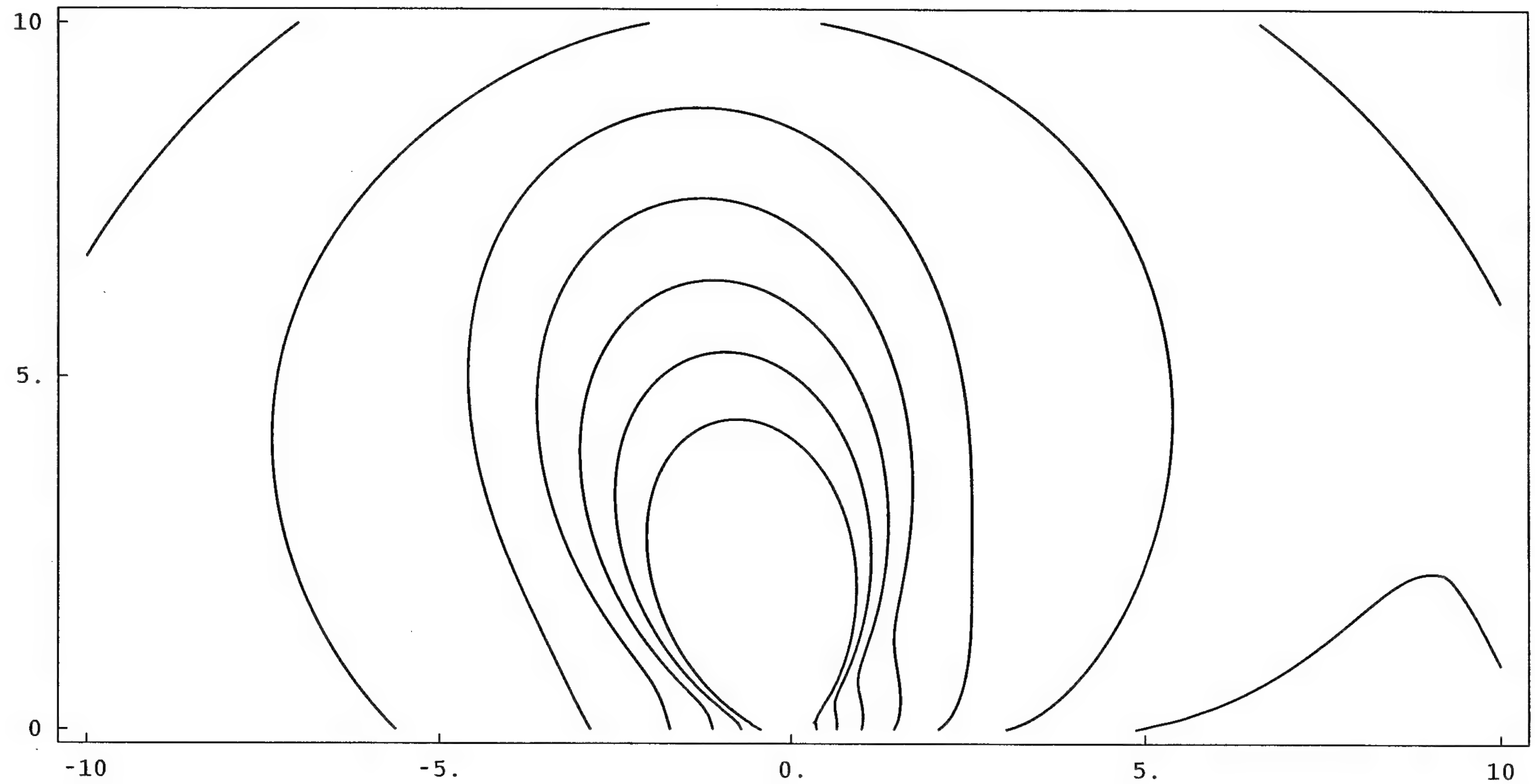

Figure 7. *P* at (5, 8, 14, 22, 37, 61, 100) psi.

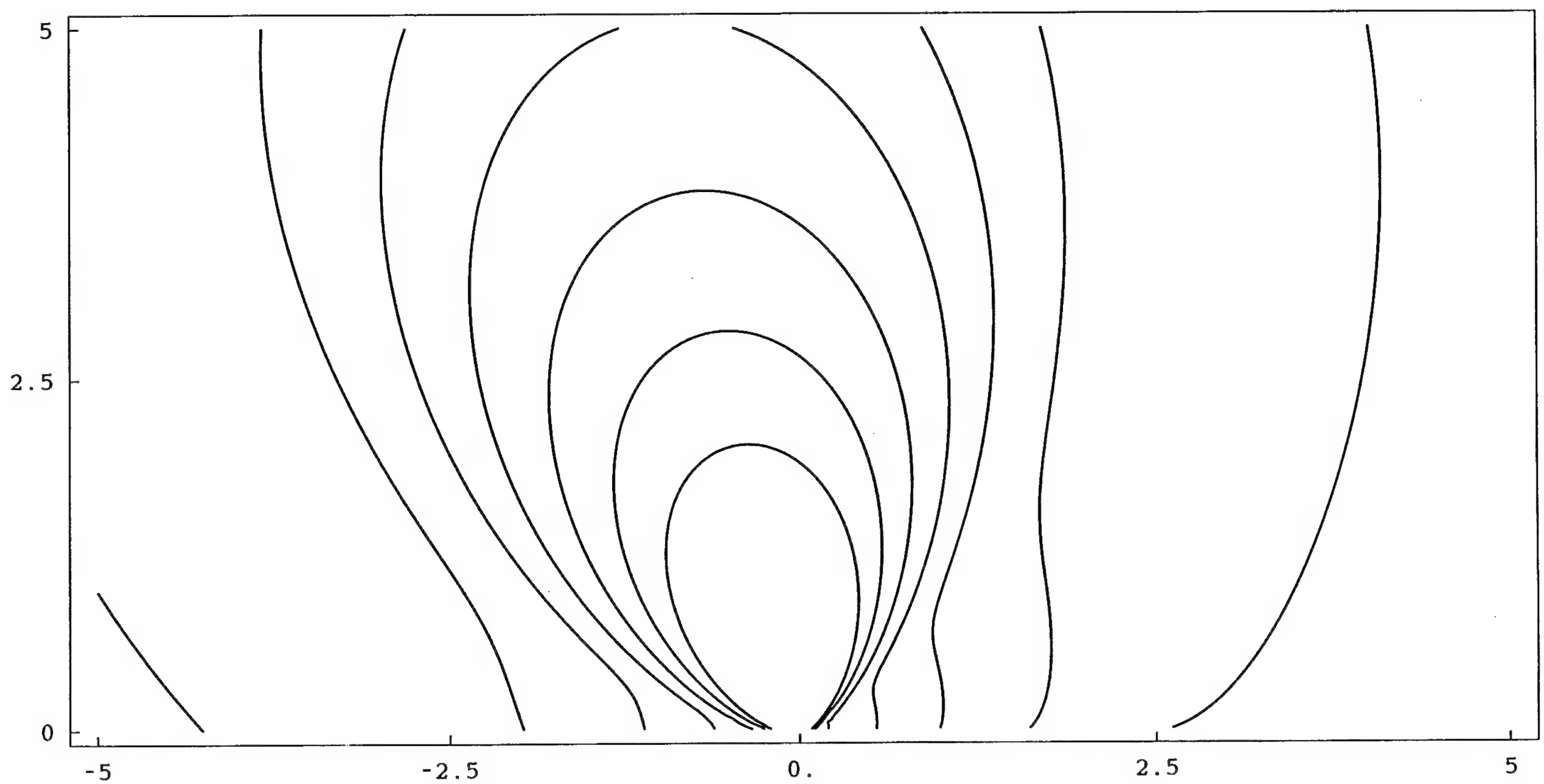

Figure 8. *P* at (10, 19, 37, 71, 136, 261, 500) psi.

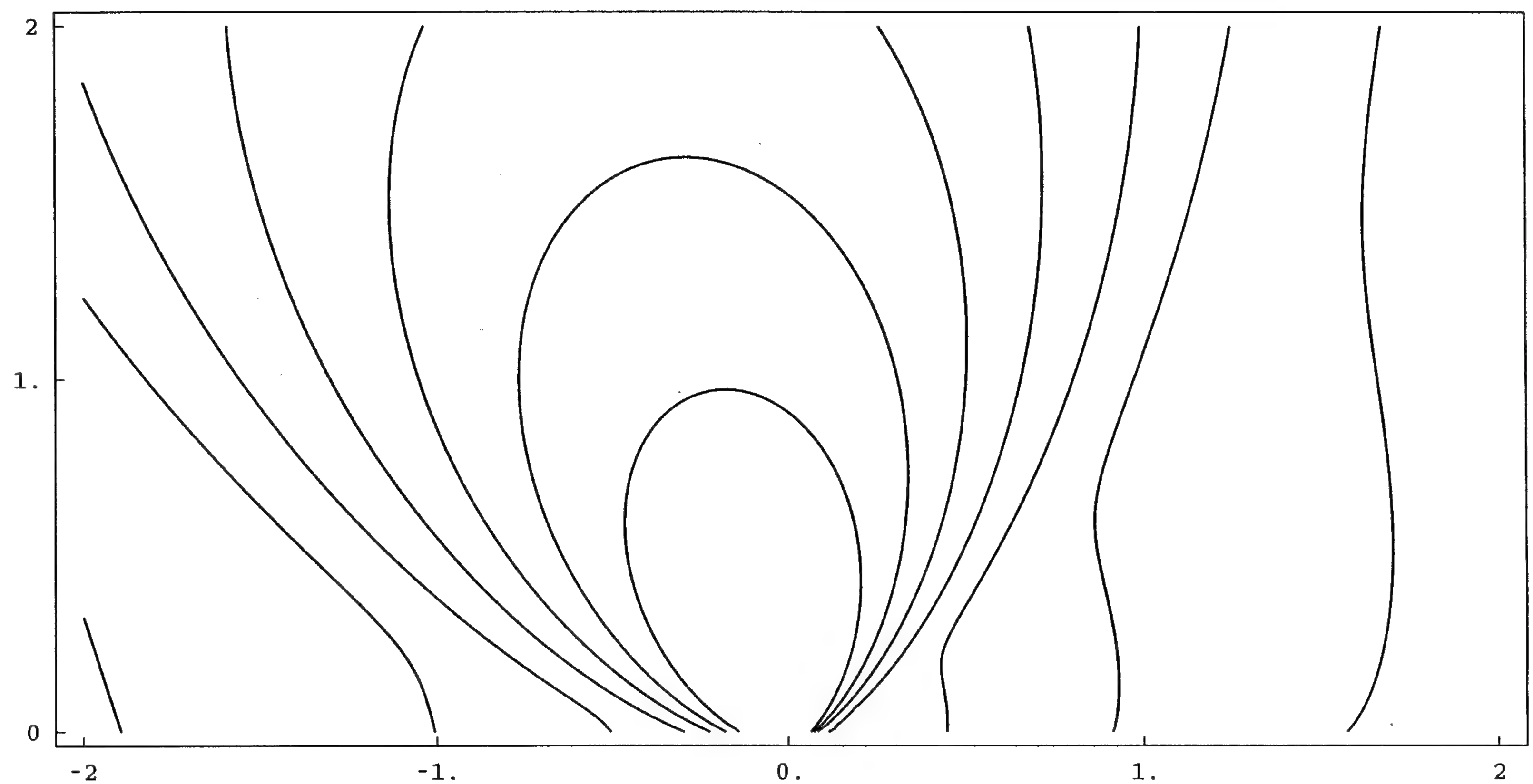

Figure 9. *P* at (20, 41, 84, 173, 356, 730, 1,500) psi.

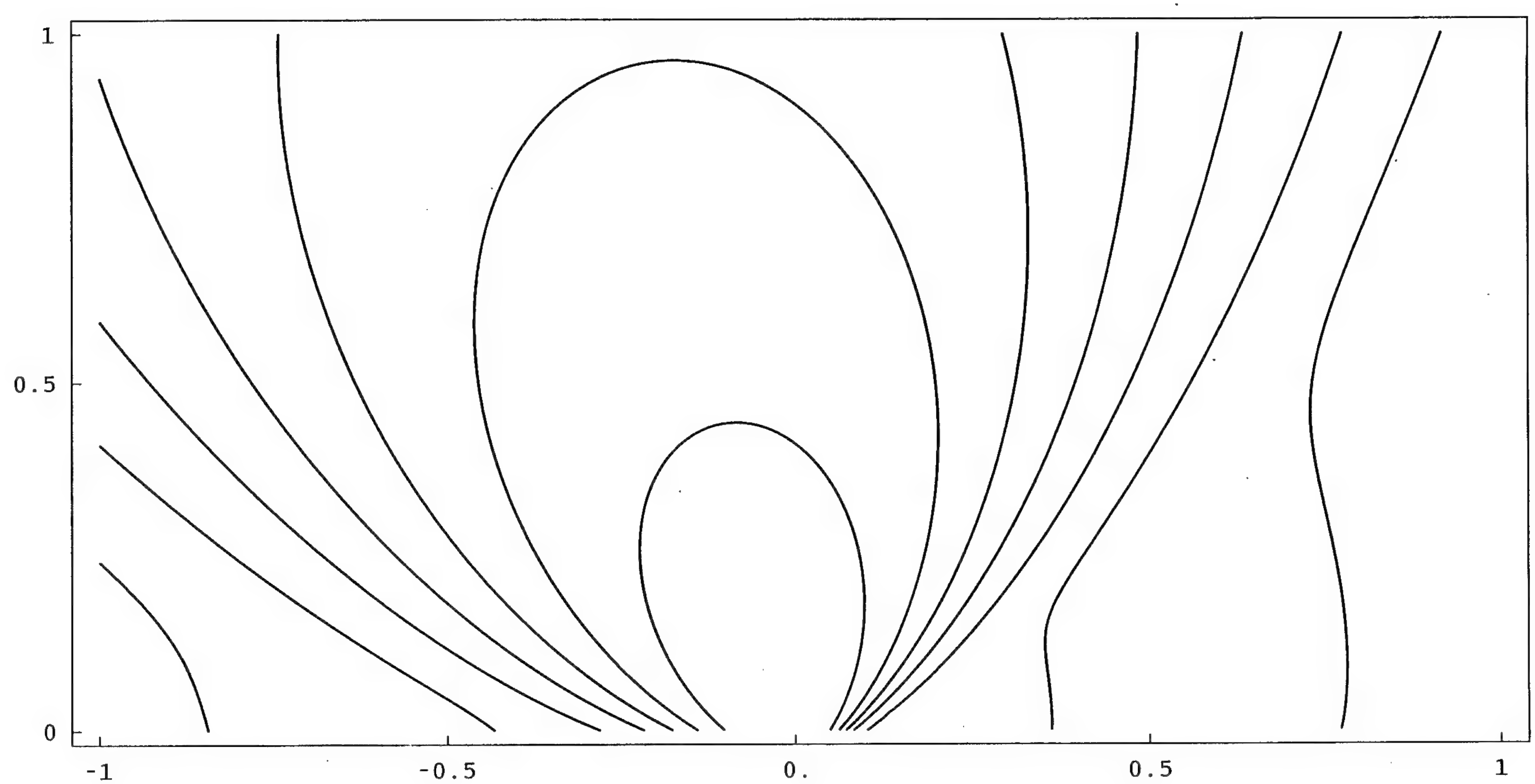

Figure 10. ***P*** **at (50, 99, 196, 387, 766, 1,516, 3,000) psi.**

6. References

1. Litt, O. Presentation at meeting. U.S. Army Research Laboratory, Aberdeen Proving Ground, MD, June 26, 1998.

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6. Wolfram, S. *Mathematica, A System for Doing Mathematics by Computer*. 2nd edition, Addison-Wesley Publishing Company, Inc., Redwood City, CA, 1991.

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Appendix: Mathematica Code

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This following Mathematica code is provided for function evaluation and visualization. This environment is useful for preliminary investigation, function selection, and experiment design. Estimation procedures are not provided. Mathematica names are generally consistent with names in the body of this report.

A.1 Evaluation. Define the utility functions `Seq` and `Lseq` to create linear and logarithmic sequences. The resulting sequences range from `a` to `b` and contain `n` elements.

```
(* UTILITY FUNCTIONS *)
Clear[Seq, Lseq];
Seq[a_, b_, n_] := Range[a, b, (b-a)/(n-1)];
Lseq[a_, b_, n_] := Exp[Seq[Log[a], Log[b], n]];
```

Define the model constants and parameters `Ao`, `Bo`, `Co`, `w`, `alpha`, `m`, `No`, `Mo`, `z0`, `z1`, and `z2`.

```
 (* PARAMETERS & CONSTANTS *)
Clear[Ao, Bo, Co, w, alpha, m, No, Mo, z0, z1, z2];
(* sphere function parameters *)
Ao = 32.97;
Bo = 3.555;
Co = 0.5;
(* charge weight and scaling exponent *)
w = 1.0;
alpha = 1/3.0;
(* model parameters *)
m = 7/4;
No = 2.0;
Mo = 4;
(* transition function parameters *)
z0 = 10; z1 = 5; z2 = 15;
```

Define the model functions `f01`, `f`, `n`, `k`, `g`, `gi`, `Psz`, and `P`.

```
(* BLAST FUNCTIONS *)
Clear[f01, n, f, s, g, gi, Psz, P];
(* basic transition function *)
f01[x_] := 1/2 + ArcTan[x]/Pi;
(* generic transition fucntion *)
f[z_, m_, r_] := f01[r(z-m)];
(* level curve exponent function *)
n[z_, No_] := No ( 1 - f[z, z0, 1]);
```

```
(* equivalent sphere weight function *)
s[z_, b_, z1_, z2_] := z/b /; z <= z1;
s[z_, b_, z1_, z2_] :=
  z/((b z2-z1)/(z2-z1) + (1-b)/(z2-z1)z) /; z1<z && z<z2;
s[z_, b_, z1_, z2_] := z /; z >= z2;


(* level curve function *)
g[z_, phi_, m_, No_, Mo_] := z Sin[m phi/2]^(4 n[z,No]/m);

(* level curve inverse function *)
gi[z_, phi_, m_, No_, Mo_] := Module[{u0, u1, u},
  If[g[z, phi, m, No, Mo]>z,
    For[u0=z, g[u0,phi,m,No,Mo] > z, u0=u0/3]; u1=3 u0,
    For[u1=z, g[u1,phi,m,No,Mo] < z, u1=3 u1]; u0=u1/3];
  u = FindRoot[g[u, phi, m, No, Mo] = = z, {u, {u0, u1}},
    MaxIterations -> 25][[1]][[2]];
  u];


(* sphere charge function *)
Psz[z_] := Exp[Ao / ( z + Bo ) - Co];


(* peak overpressure model function *)
P[x_, y_] := Module[{r, phi, z, u0, u1, u, P},
  r = Sqrt[x^2+y^2];
  phi = ArcTan[x, y];
  z = r/w^alpha;
  u = gi[z, phi, m, No, Mo];
  P = f[z,z0,1]Psz[z]+(1-f[z,z0,1])Psz[s[u,Mo^alpha,z1,z2]];
  P]
```

A.2 Visualization. Define the function `Ptable` to evaluate `P(x, y)` on an `nx` by `ny` grid with `-X1` ⩽ `x` ⩽ `X1` and `dy` ⩽ `y` ⩽ `X1+dy`.

```
Ptable[X1_, nn_] := Module[
  {nx, ny, dy = 0.05, X0, Y0, X, Y, Y1},
  X0 = -X1; Y0 = 0; Y1 = X1; nx = nn; ny = nn/2;
  X = Seq[X0, X1, nx];
  Y = dy+Seq[Y0, Y1, ny];
  XP = Table[P[X[[i]], Y[[j]]], {i, nx}, {j, ny}];
  {X0, X1, nx, Y0, Y1, ny, XP}]
```

Define the function `Pshow` to graph `XP`, the result of `Ptable`. The other arguments are the lowest contour level (`L0`), the highest contour level (`L1`), and the number of countour levels (`NL`).

```
Pshow[XP_, L0_, L1_, NL_] := Module[
  {X0, X1, nx, Y0, Y1, ny, Ftix, Mhue, Clevels, XF},
  X0 = XP[[1]]; X1 = XP[[2]]; nx = XP[[3]];
  Y0 = XP[[4]]; Y1 = XP[[5]]; ny = XP[[6]];
  PP = XP[[7]];
  Mhue[h_] = Hue[1, 0, 1];
  Ftix = {{{1, X0}, {0.75 1 + 0.25 nx, 0.75 X0 + 0.25 X1},
           {0.5 1 + 0.5 nx, 0.5 X0 + 0.5 X1},
           {0.25 1 + 0.75 nx, 0.25 X0 + 0.75 X1}, {nx, X1}},
          {{1, Y0}, {0.5 1 + 0.5 ny, 0.5 Y0 + 0.5 Y1},
           {ny, Y1}}, None, None};
  Clevels = Log[Lseq[L0, L1, NL]];
  Print[N[Round[1 Exp[Clevels]] / 1]];
  XF = ListContourPlot[Transpose[Log[PP]],
         AspectRatio -> 1/2, ColorFunction -> Mhue,
         FrameTicks -> Ftix, Contours -> Clevels,
         ContourSmoothing -> 32];
  XF]
```

A.3 Example Use. The graphics in this report were produced by the following commands. First, create the numerical arrays. With `nn`= 200, the functions are evaluated on a 200 $\times$ 100 grid. This takes a while. Smaller values of `nn` can be used for quicker, lower resolution results.

```
nn = 200;
XP1 = Ptable[20, nn];
XP2 = Ptable[10, nn];
XP3 = Ptable[5, nn];
XP4 = Ptable[2, nn];
XP5 = Ptable[1, nn];
```

Then, construct the graphs.

```
XF1 = Pshow[XP1, 2, 20, 7]
XF2 = Pshow[XP2, 5, 100, 7]
XF3 = Pshow[XP3, 10, 500, 7]
XF4 = Pshow[XP4, 20, 1500, 7]
XF5 = Pshow[XP5, 50, 3000, 7]
```

Finally, export the graphics files.

```
IS = {600, 600};
Display["xf1.ps", XF1, "EPS", ImageSize -> IS];
Display["xf2.ps", XF2, "EPS", ImageSize -> IS];
Display["xf3.ps", XF3, "EPS", ImageSize -> IS];
Display["xf4.ps", XF4, "EPS", ImageSize -> IS];
Display["xf5.ps", XF5, "EPS", ImageSize -> IS];
```

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Overpressure time history data from warhead blast experiments yield peak overpressure P as a function of spatial position.

Dr. Owen Litt has proposed a model for P based on the peak-overpressure characteristics of a bare spherical charge. The direction-independent peak-overpressure function for a bare spherical charge is modified to have nonspherical level-surface structure by specifying surfaces of constant peak overpressure. This introduces a directional component into the model.

In this report, the original formulation is refined and generalized and a mathematical model and computer code are presented to evaluate the function. Such a computational device is required for model parameter estimation and experiment design.

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